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## ABSTRACT (Maximum 200 words)

This report presents a hybrid state-space self-tuner using a new dual-rate sampling scheme for digital adaptive control of continuous-time uncertain linear systems. A state-space-based recursive least-squares algorithm, together with a variable forgetting factor, is used for direct estimation of both the equivalent discrete-time uncertain linear system parameters and associated discrete-time state of a continuous-time uncertain linear system from the sampled input and output data. An analogue optimal regional pole-placement design method is used for designing an optimal observer-based analogue controller. A suboptimal observer-based digital controller is then designed from the designed analogue controller using digital redesign technique. To enhance the robustness of parameter identification and state estimation algorithms, a dynamic bound for a class of uncertain bilinear parameters and a fast-rate digital controller are developed at each fast-sampling period. Also, to accommodate computation loads and computation delay for developing the advance hybrid self-tuner, the designed analogue controller and observer gains are both updated at each slow-sampling period. This control technique has been successfully applied to benchmark control problems.

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# **Robust Hybrid State-space Self-tuning Control Using Dual-rate Sampling**

## **Summary of research results**

Most flight vehicles and industrial control processes can be approximately described by uncertain linear models. The uncertainty in these systems arises from unmodeled dynamics, parameter variations, etc. Also, these uncertain linear models operated in real environments constantly face parameter and/or structural variations owing to the changes in operating conditions, disturbances acting on the system, etc. During the last three years, we have investigated the robust adaptive control of these uncertain systems. The main research accomplishments are summarized as follows:

1. For robust control of the uncertain systems, we have recently developed a new observer-based robust- $H_\infty$  control law [14, 18] and successfully applied it to the benchmark problem which is a simplified version of both the rigid body mode and the flexible body mode with uncertainty. Our research result was chosen as one of the eleven selected papers published in the special section of "Robust Control Design for a Benchmark Problem" published in AIAA J. of Guidance, control, and Dynamics, Vol. 15, No.5, Sept.-Oct. 1992.
2. With the rapid advances in digital technology and computers, the designed continuous-time state-feedback controller is often required to be implemented using digital controller for better reliability, less cost, more flexibility, and better performance of the digitally controlled continuous-time systems. The process of converting a continuous-time controller of a continuous-time system into an equivalent discrete-time controller is called digital redesign. During the last three years, we have successfully developed various new digital redesign techniques [6, 7, 10, 12, 20]. We believe that our research results will give state-of-the-art digital controllers for use in hybrid control of sampled-data systems.
3. For robust adaptive control of the afore-mentioned uncertain systems, we have recently developed a new hybrid state-space self-tuner using dual-rate sampling scheme for digital adaptive control of continuous-time uncertain systems [1, 23] and successfully applied it to the above-mentioned benchmark problem.
4. To perform effective digital simulation, parameter identification, and hybrid controller design of uncertain systems, we have recently developed a new technique for model conversions of a continuous-time (discrete-time) structured uncertain system to an equivalent discrete-time (continuous-time) uncertain model [25]. We believe that our research results will make significant contribution to the analysis and design of both continuous-time and discrete-time uncertain systems.

Based on the research results during the period from May 1, 1991 to August 31, 1994, twenty five technical papers have been published in renowned journals. They are listed as follows:

1. Leang S. Shieh, Xiao M. Zhao and John W. Sunkel, "Hybrid State-space Self-tuning Control Using Dual-rate Sampling," IEE Proceedings - D, Vol.138, No.1, pp.50-58, 1991

Abstract

This paper presents a hybrid state-space self-tuning control scheme using dual-rate sampling for suboptimal digital adaptive control of linear time-invariant continuous-time multivariable stochastic systems with unknown parameters. An equivalent fast-rate discrete-time state-space innovation model (with estimated states) of the continuous-time system is constructed by using the estimated system parameters and Kalman gain. To utilise the existing optimal regional-pole assignment method developed in the continuous-time domain, the constructed fast-rate discrete-time model is converted into an equivalent continuous-time model for the development of a state-feedback optimal control law with pole placement in a specific region. The developed analogue optimal control law is then converted into an equivalent pseudo-slow-rate digital control law via the proposed digital redesign technique, which can be realised via slow-rate digital electronics. The proposed method enables the development of a digitally implementable advanced control algorithm for digital adaptive control of continuous-time multivariable stochastic systems which may be unstable and/or have nonminimum phase.

2. Sekar Ganesan, Leang S. Shieh and Mohanad M. Mehio, "Sequential Design of Linear Quadratic State Regulators with Prescribed Eigenvalues and Specified Relative Stability," Computers Math. Applic. Vol.21, No.4, pp.1-10, 1991

Abstract

A digital redesign technique is developed for determining the digital version of an optimal momentum management controller previously designed by the authors for the Space Station Freedom. The technique matches the continuous-time and discrete-time states at all sampling instants to find a pseudo-continuous-time quadratic regulator from a continuous-time quadratic regulator. It is shown that the digital redesigned states closely match the continuous-time optimal states. It is also shown that the digital redesigned state-feedback control law based on the bilinear transformation method is a class of the proposed control law. The digital redesign technique is then extended to find the digital version of the continuous-time optimal observer. It is shown that the states of the redesigned digital observer closely match those of the continuous-time optimal observer.

3. Chyi Hwang, Tong-Yi Guo and Leang-San Shieh, "A Canonical State-space Representation for SISO Systems Using Multipoint Jordan CFE," Journal of the Franklin Institute Vol.328, No. 2/3, pp. 207-216, 1991.

Abstract

A canonical state-space realization based on the multipoint Jordan continued-fraction expansion (CFE) is presented for single-input-single-output (SISO) systems. The similarity transformation matrix which relates the new canonical form to the phase-variable canonical form is also derived. The presented canonical state-space representation is particularly attractive for the application of SISO system theory in which a reduced-dimensional time-domain model is necessary.

4. J.W.Sunkel, L.S.Shieh, "Multi-stage Design of An Optimal Momentum Management Controller for the Space Station," AIAA J. of Guidance, Control, and Dynamics, Vol.14, No.3, pp.492-502,1991

Abstract

This paper presents a multi-stage design scheme for determining an optimal control moment gyro momentum management and attitude control system for the Space Station Freedom. First, the Space Station equations of motion are linearized and then block-decomposed into two block- decoupled subsystems using the matrix sign algorithm. Next, a sequential design procedure is utilized for designing a linear quadratic regulator for each subsystem, which optimally places the eigenvalues of the closed-loop subsystem in the region of an open sector, bounded by lines inclined at  $\pm\pi/2k$  (for  $k = 2$  or 3) from the negative real axis, and the left-hand side of a line parallel to the imaginary axis in the s-plane. Simulation results are presented to compare the resultant designs.

5. Chyi Hwang, Ming-Jeng Lu, and Leang S. Shieh, "Improved FFT-based Numerical Inversion of Laplace Transforms via Fast Hartley Transform Algorithm," Computers Math. Applic., Vol.22, No.1, pp.13-24, 1991

Abstract

The disadvantages of numerical inversion of the Laplace transform via the conventional fast Fourier transform (FFT) are identified and an improved method is presented to remedy them. The improved method is based on introducing a new integration step length  $\Delta\omega = \pi/mT$  for trapezoidal-rule approximation of the Bromwich integral, in which a new parameter  $m$ , is introduced for controlling the accuracy of the numerical integration. Naturally, this method leads to multiple sets of complex FFT computations. A new inversion formula is derived such that  $N$  equally-spaced samples of the inverse Laplace transform function can be obtained by  $[m/2] + 1$  sets of  $N$ -point complex FFT computations or by  $m$  sets of real fast Hartley transform (FHT) computations.

6. Leang S. Shieh and Jian L. Zhang N.P. Coleman "Optimal Digital Redesign of Continuous time Controllers," Computers Math. Applic., Vol.22, No.1, pp.25-35,1991

Abstract

This paper proposes a new optimal digital redesign technique for finding a dynamic digital control law from the available analog counterpart and simultaneously minimizing a quadratic performance index. The proposed technique can be applied to a system with a more general class of reference inputs, and the developed digital regulator can be implemented using low cost microcomputers.

7. Sunkel, J.W., L.S. Shieh, and J.L. Zhang, "Digital Redesign of an Optimal Momentum Management Controller for the Space Station," AIAA Journal of Guidance, Control and Dynamics, Vol. 14, No. 4, pp. 712-723, July-August 1991.

Abstract

A digital redesign technique is developed for determining the digital version of an optimal momentum management controller previously designed by the authors for the Space Station Freedom. The technique matches the continuous-time and discrete-time states at all sampling instants to find a pseudocontinuous-time quadratic regulator from a continuous-time

quadratic regulator. It is shown that the redesigned digital states closely match the continuous-time optimal states. It is also shown that the digital redesigned state-feedback control law based on the bilinear transformation method is a class of the proposed control law. The digital redesign technique is then extended to find the digital version of the continuous-time optimal observer. It is shown that the states of the redesigned digital observer closely match those of the continuous-time optimal observer.

8. Shieh, L.S., Z. Liu and J.W. Sunkel, "Optimal Uniform-Damping Ratio Controller for Sequential Design of Multivariable systems," *International Journal of Systems Sciences*, Vol. 22, No. 8, pp. 1371-1389, 1991.

### Abstract

An optimal uniform-damping ratio controller is developed for the sequential design of a multivariable control system so that the designed closed-loop poles of the respective multivariable system and reduced-order observer are exactly placed on the negative real axis and/or the boundaries of desired sectors with constant-damping ratios. The functions in the quadratic performance index to be minimized are chosen as a combination of the weighted outputs, reduced states and inputs. Also, the optimal uniform-damping ratio controller is a combination of optimal output feedback and optimal reduced-order state-feedback controllers. A numerical example is given to demonstrate the design procedure.

9. Tsai, J.S.H., C.M. Chen, and L.S. Shieh, "digital Modelling, Ideal State Reconstructor, and Control for Time-delay Sampled-data Systems," *Applied Mathematical Modelling*, Vol. 15, No. 11 and 12, pp. 576-585, December 1991.

### Abstract

The cascaded discrete-time state-space representation of a cascaded continuous-time system with fractional input delays is established. Based on the time-delay digital modelling, a practically implementable ideal state reconstructor is also established such that system states are exactly reconstructed via the measurement histories of inputs and outputs without a state observer. By utilizing the block-pulse function approximation the digital modelling of cascaded continuous-time systems with fractional input delays can be carried out, and an artificial input design method is proposed to determine the state feedback again. Thus the practically implementable digital control law can be established for digital control of time-delay sampled-data systems. An illustrative example is shown to demonstrate the effectiveness of the proposed method.

10. Shieh, L.S., B.B. Decrocq and J.L. Zhang, "Optimal Digital Redesign of Cascaded Analogue Controllers," *Optima Control Applications and Methods*, Vol. 12, pp. 205-219, 1991.

### Abstract

This paper presents a new, optimal digital redesign technique for finding an optimal cascaded digital controller from the given continuous-time counterpart by minimizing a quadratic performance index. The control gains can be obtained by solving a set of Lyapunov equations. The developed optimal cascaded digital controller enables the state and/or outputs of the digitally controlled closed-loop sampled-data system to optimally match those of the original continuous-time closed-loop system at any instant between sampling periods. The developed control law can be implemented using inexpensive and reliable digital electronics with a relatively long sampling period.

**11.** Zhao, X.M., L.S. Shieh, J.W. Sunkel, and Z.Z. Yuan, "Self-turning Control of Attitude and Momentum Management for the Space Station," AIAA Journal of Guidance Control and Dynamics, Vol. 15, No. 1, pp. 17-27, January-February 1992.

### **Abstract**

This paper presents a hybrid state-space self-tuning design methodology using dual-rate sampling for suboptimal digital adaptive control of attitude and momentum management for the Space Station. This new hybrid adaptive control scheme combines an on-line recursive estimation algorithm for indirectly identifying the parameters of a continuous-time system from the available fast-rate sampled data of the inputs and states and a controller synthesis algorithm for indirectly finding the slow-rate suboptimal digital controller from the designed optimal analog controller. The proposed method enables the development of digitally implementable control algorithms for the robust control of Space Station Freedom with unknown environmental disturbances and slowly time-varying dynamics.

**12.** Shieh, L.S., J.L. Zhang and J.W. Sunkel, "A New Approach to the Digital Redesign of Continuous-time Controllers," Control Theory and Advanced Technology, Vol. 8, No. 1, pp. 37-57, March 1992.

### **Abstract**

This paper presents a new method for digital redesign of both a continuous-time state-feedback control law and a cascaded analog controller. The concept of the Law of Mean from the integral calculus is utilized for the development of the equivalent digital controllers from the available analog controllers. A tuning parameter has been introduced into the digitally redesigned control gains so that the digitally controlled states will closely match the original continuous-time states. The proposed digital controllers give excellent performance.

**13.** Guo, T.Y., C. Hwang, L.S. Shieh, and C.H. Chen, "Reduced-order Models of a 2-D Linear Discrete Separable-Denominator System Using Bilinear Routh Approximations," Proceedings of the IEE (England) Part G. Electronic Circuits and Systems, Vol. 139, No. 1, pp. 45-56, February 1992.

### **Abstract**

The paper extends the Routh approximation method for one-dimensional (1-D) discrete systems to two-dimensional (2-D) discrete systems for finding stable reduced-order models from a stable high-order 2-D linear discrete separable-denominator system (SDS). The extension is achieved by exploring new properties of the 1-D Routh canonical model and establishing new 2-D bilinear Routh canonical models. Without explicitly performing bilinear transformations, a computationally-efficient procedure is presented for finding the bilinear Routh reduced-order models. The properties of the obtained 2-D bilinear Routh approximants are discussed in detail. In addition, a new 2-D bilinear Routh canonical state-space realisation is presented from which the low-dimensional state-space models corresponding to the bilinear Routh approximants can be obtained by a direct truncation procedure. Furthermore, the relationships among the states of the bilinear Routh reduced-dimension model, the aggregated model, and the original system are explored. Numerical examples are given to demonstrate the effectiveness of the proposed method.

**14.** Wang, Y.J., L.S. Shieh and J.W. Sunkel, "Robust Stabilization, Robust Performance and Disturbance Attenuation for Uncertain Linear Systems," Computers and Mathematics with Applications, Vol. 23, No. 11, pp. 67-80, 1992.

### Abstract

This paper presents a linear quadratic approach to the robust stabilization, robust performance, and disturbance attenuation of uncertain linear systems. The state-feedback designed systems provide both robust stability with optimal performance and disturbance attenuation with  $H_\infty$ -norm bounds. The proposed approach can be applied to *matched* and/or *mismatched* uncertain linear systems. For a matched uncertain linear system, it is shown that the disturbance-attenuation robust-stabilizing controllers with or without optimal performance always exist and can be easily determined without searching; whereas, for a mismatched uncertain linear system, the introduced tuning parameters greatly enhance the flexibility of finding the disturbance-attenuation robust-stabilizing controllers.

15. Shieh, L.S., G. C. Chen and J.S.H. Tsai, "Hybrid Suboptimal Control of Multi-rate Multi-loop Sampled-data Systems," *International Journal of Systems and Science*, Vol. 23, No. 6, pp. 839-854, 1992.

### Abstract

A hybrid state-space control scheme for suboptimal digital control of a cascaded continuous-time system using dual rate sampling is presented. First, an optimal regional-pole placement technique is utilized to find an optimal state-feedback control law for a subsystem connected in the linear loop of the overall system. Next, the designed analogue control law is converted into an equivalent fast-rate digital control law using the recently developed digital redesign technique. Then, the digitally redesigned subsystem is converted into an equivalent continuous-time model. As a result, the overall continuous-time model can be formulated from the converted subsystem and the rest of the analogue subsystems to be designed. Moreover, the optimal regional-pole placement technique is applied again to the overall continuous-time model in order to obtain the overall analogue state-feedback control law. Finally, the digital redesign technique is employed again to convert the overall analogue control law obtained to an equivalent slow-rate digital control law. For practical implementations of the developed digital control laws with various sampling rates, the existing ideal state reconstructor method is redeveloped to construct the ideal discrete-time states using multi-rate input-output data. A practical semi-active terminal homing missile is used as an illustrative example to demonstrate the proposed design method.

16. Lee, Y.C., C. Hwang and L.S. Shieh, "Order Reduction of z-transfer Functions Via Multipoint Jordan Continued-Fraction Expansion," *Journal of the Franklin Institute*, Vol. 329, No. 3, pp. 583-590, May 1992.

### Abstract

The order reduction problem of z-transfer functions is solved by using the multipoint Jordan continued-fraction expansion (MJCCE) technique. An efficient algorithm that does not require the use of complex algebra is presented for obtaining an MJCCE from a stable z-transfer function with expansion points selected from the unit circle and/or the positive real axis of the z-plane. The reduced-order models are exactly the multipoint Pade approximants of the original system and, therefore, they match the (weighted) time-moments of the impulse response and preserve the frequency responses of the system at some characteristic frequencies, such as gain crossover frequency, phase crossover frequency, bandwidth, etc.

17. Tsai, J.S.H., C.M. Chen, and L.S. Shieh, "A Computer-aided Method for Solvents and Spectral Factors of Matrix Polynomials," *Applied Mathematics and Computation*, Vol. 47, No. 2 and 3, pp. 211-235, February 1992.

### Abstract

A new approach for determining the complete sets of solvents and spectral factors of a monic matrix polynomial is proposed. A systematic method for determining the initial guess for the extended multidimensional Newton-Raphson method is first proposed, such that the eigenspectrum corresponding to each solvent of the matrix polynomial can be determined. With the evaluated eigenspectra, complete sets of solvents and spectral factors of a monic matrix polynomial can be obtained by utilizing the applications and the advantages of the principal-nth-root method, the matrix sign function, and the block-power method. The established algorithms can be applied in the analysis and/or design of systems described by high-degree vector differential equations and/or matrix fractions.

18. Wang, Y.J., L.S. Shieh and J.W. Sunkel, "Observer-based Robust- $H_\infty$  Control Laws for Uncertain Linear Systems," *AIAA Journal of Guidance Control and Dynamics, Special Section: Robust Control Design for a Benchmark Problem*, Vol. 15, No. 5, pp. 1125-1133, September-October, 1992.

### Abstract

Based on the algebraic Riccati equation approach, this paper presents a simple, flexible method of designing full-order observer-based robust- $H_\infty$  control laws for linear systems with structured parameter uncertainty. The observer-based robust- $H_\infty$  output-feedback control law, obtained by solving three augmented algebraic Riccati equations, provides both robust stability and disturbance with  $H_\infty$ -norm bound for the closed-loop uncertain linear system. Several tuning parameters are embedded into the augmented algebraic Riccati equations so that flexibility in finding the symmetric positive-definite solutions (and hence the robust- $H_\infty$  control law  $s$ ) is significantly increased. A benchmark problem associated with a mass-spring system, which approximates the dynamics of a flexible structure, is used to illustrate the design methodologies, and simulation results are presented.

19. Tsai, J.S.H., S.S. Chen, and L.S. Shieh, "On the Singular case of Jury's Algorithms for Stability Test of Linear Discrete systems," *IMA Journal of Mathematical Control and Information*, Vol. 9, pp. 231-244, 1992.

### Abstract

For a stability test of linear discrete systems in a tabular form, two singular cases of Jury's algorithms are considered, in which a row with some (but not all) vanishing leading elements and a row with all zero elements arise respectively. For the singular case of rows with some (but not all) vanishing leading elements, Yeung's method is improved for efficient usages. Based on the newly improved algorithm in treating all-zero rows, the number of roots on the unit circle and their respective orders can be determined. As a result, the situation of conditional stability or instability can be distinguished by the criteria developed in this paper.

20. Tsai, J.S.H., L.S. Shieh and J.L. Zhang, "An Improvement of the Digital Redesign Method Based on the Block-pulse Function Approximation," *Circuits, Systems, and Signal Processing*, Vol. 12, No. 1, pp. 37-49, 1993.

### Abstract

This paper proposes an effective method to improve the digital redesign method via the block-pulse function approach. The coefficients of the block-pulse function expansion are exactly evaluated such that the desired digitally redesigned feedback gain and forward gain will be obtained. A numerical example is given to demonstrate the effectiveness of the proposed method.

**21.** Tsai, J.S.H., C.M. Chen, and L.S. Shieh, "Modelling of Multi-rate Feedback Systems Using Uniform-rate Models," Applied Mathematical Modeling, Vol. 17, No. 1, pp. 2-14, 1993.

### **Abstract**

A new methods is proposed to analyze a multirate feedback system by reducing it to an equivalent uniform-rate feedback system. The solution is essentially based on the developments of state-space model conversions in closed-loop systems with multiple rates. Either a slow-rate controlled system with fast-rate feedback or a fast-rate controlled system with a slow-rate feedback is modeled by state-space expressions with either a unified slow-rate frame or a uniform-rate frame. It is shown that the proposed method is suitable for computer programming and convenient for analysis and design of multirate sampled-data control systems.

**22.** Tsai, J.S.H., T. Chu, and L.S. Shieh, "The optimal Regional-pole-placement Design Method for Singular Systems," Journal of Control Systems and technology, Vol. 1, No. 1, pp. 61-73, 1993.

### **Abstract**

This paper presents a simple and efficient method for decomposition of a singular system into a reduced-order regular subsystem and nondynamic subsystem. It also develops an efficient method for eliminating all impulsive modes from the singular system. As a result, the optimal regional-pole-placement design method developed for regular system can be applied to singular system.

**23.** L.S. Shieh, Y.J. Wang, and J.W. Sunkel, "Hybrid State-space Self-tuning Control of Uncertain Linear Systems," Proceeding of IEE (England) Part D. Control Theory and Application, Vol. 140, No. 2, pp. 99-110, March 1993.

### **Abstract**

This paper presents a hybrid state-space self-tuner using a new dual-rate sampling scheme for digital adaptive control of continuous-time uncertain linear systems. A state-space-based recursive least-squares algorithm, together with a variable forgetting factor, is used for direct estimation of both the equivalent discrete-time uncertain linear system parameters and associated discrete-time state of a continuous-time uncertain linear system from the sampled input and output data. An analogue optimal regional pole-placement design method is used for designing an optimal observer-based analogue controller. A suboptimal observer-based digital controller is then designed from the designed analogue controller using digital redesign technique. To enhance the robustness of parameter identification and state estimation algorithms, a dynamic bound for a class of uncertain bilinear parameters and a fast-rate digital controller are developed at each fast-sampling period. Also, to accommodate computation loads and computation delay for developing the advance hybrid self-tuner, the designed analogue controller and observer gains are both updated at each slow-sampling period. This control technique has been successfully applied to benchmark control problems.

**24.** Chen, Z.Q., Z.Z. Yuan, and L.S. Shieh, "Global Convergence of Multivariable State-space Pole-assignment Self-tuning Scheme," Control Theory and Advanced Technology, Vol.9, No.4, pp.909-923, 1993

### **Abstract**

This paper presents an analysis of stability and convergence of a state-space pole-assignment self-tuning scheme and establishes the global convergence of the scheme.

**25. Shieh, L.S., J. Gu, and Y.L. Bao, "Model Conversions of Uncertain Linear Systems Using the Pade and Inverse-Pade Method," *Proceeding of IEE(England) Part D. Control Theory and Applications*, Vol. 140, No.6, pp.455-464, November, 1993.**

### **Abstract**

This paper presents a mixed method consisting of the Pade approximation and the inverse-Pade approximation method for model conversions of a continuous-time (discrete-time) structured uncertain linear system to an equivalent discrete-time (continuous-time) structured uncertain linear model. This allows the use of well-developed theorems and algorithms in the discrete-time (continuous-time) domain to indirectly solve the continuous-time (discrete-time) domain problems. When the ratio of the uncertainties ( $\Delta A, \Delta B$ ) to the nominal system parameters

( $A_0, B_0$ ) and/or the sampling period ( $T \leq \frac{2}{\|A_0\| + \|\Delta A\|}$ ) are sufficiently small, the linearized

uncertain models can be obtained by neglecting the nonlinear uncertain terms,  $O(T^2 \Delta A^2)$  and  $O(T^2 \Delta A \Delta B)$ , in the state-space models. The comparison of a high-order Pade approximate model with the bilinear approximate model and the Taylor-series approximate model is also presented. Moreover, this paper also presents a method to estimate tighter bounds of discrete-time (continuous-time) structured uncertainties based on the given continuous-time (discrete-time) structured uncertainties.

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